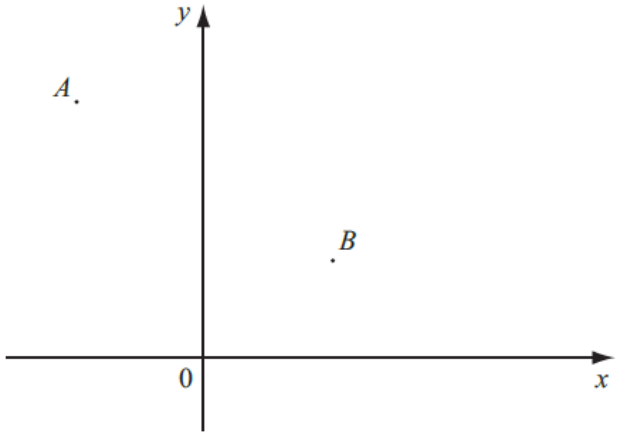
6 – Vectors

Student name: Answers Score: _____

1. The points $A(-3, 5)$ and $B(3, 2)$ are shown on the diagram below.



NOT TO SCALE

(a) (i) Write down the vector $\overrightarrow{AB}$ in component form.

$\begin{pmatrix} 6 \\ -3 \end{pmatrix}$ [2]

(ii) Find $|\overrightarrow{AB}|$ leaving your answer in surd form.

$3\sqrt{5}$ [2]

(b) Calculate the gradient of the line AB.

$\frac{-3}{6}$ [2]

(c) Calculate the co-ordinates of the midpoint of the line AB.

(0 , 3.5) [1]

(d) Find the equation of the perpendicular bisector of the line AB.

$y = 2x + 3.5$ [2]

2. $\mathbf{p} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$

(a) Write $2\mathbf{p} - \frac{1}{2}\mathbf{q}$ as a column vector.

$\begin{pmatrix} 12 \\ 1 \end{pmatrix}$ [2]

(b) Find $|\mathbf{q}|$ leaving your answer in surd form.

$2\sqrt{5}$ [2]

3. $\mathbf{p} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $\mathbf{q} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$

(a) Find $2\mathbf{p} - 3\mathbf{q}$.

$\begin{pmatrix} 16 \\ -3 \end{pmatrix}$ [2]

(b) Calculate $|\mathbf{q}|$.

5 [2]

4. $\mathbf{p} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ $\mathbf{q} = \begin{pmatrix} 5 \\ -7 \end{pmatrix}$

(a) Find $\mathbf{p} + \mathbf{q}$.

$\begin{pmatrix} 3 \\ -4 \end{pmatrix}$ [2]

(b) Work out $|\mathbf{p} + \mathbf{q}|$.

5 [2]

5. $\mathbf{p} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $\mathbf{q} = \begin{pmatrix} -3 \\ 5 \end{pmatrix}$

Find

(a) $2\mathbf{p} - 3\mathbf{q}$,

$\begin{pmatrix} 13 \\ -9 \end{pmatrix}$ [2]

(b) $|\mathbf{p}|$.

$\sqrt{13}$ [2]

6. Find the magnitude of $\begin{pmatrix} -6 \\ 4 \end{pmatrix}$.

Write your answer in surd form as simply as possible.

$2\sqrt{13}$ [3]

7. $\mathbf{a} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ $\mathbf{b} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$

When $2\mathbf{a} + k\mathbf{b} = \begin{pmatrix} -2 \\ 16 \end{pmatrix}$ find the value of k.

$k = 4$ [3]

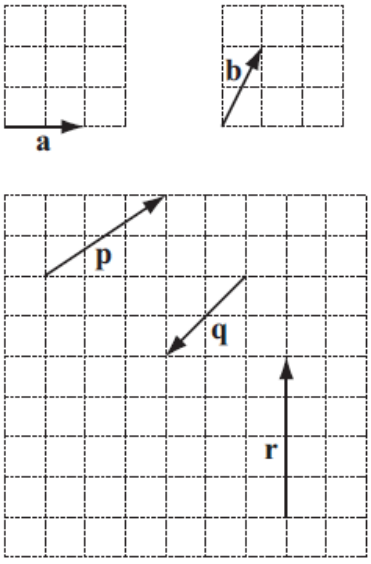
8. The line $2x + 3y = 36$ intersects the x-axis at P and the y-axis at Q.
M is the midpoint of PQ.

Find the column vector $\overrightarrow{OM}$ where O is the origin.

$\begin{pmatrix} 9 \\ 6 \end{pmatrix}$

[4]

9.



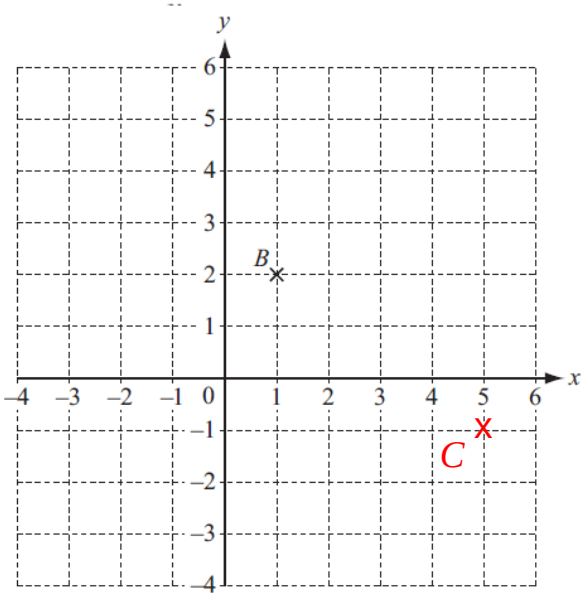
Write the vectors **p**, **q** and **r** in terms of **a** and **b**.

p = **a + b**

q = **$-\frac{1}{2}$ a - b**

r = **2b – a** [3]

10. **(a)**



0

7

B is the point $(1, 2)$ and $\overrightarrow{BC} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$.

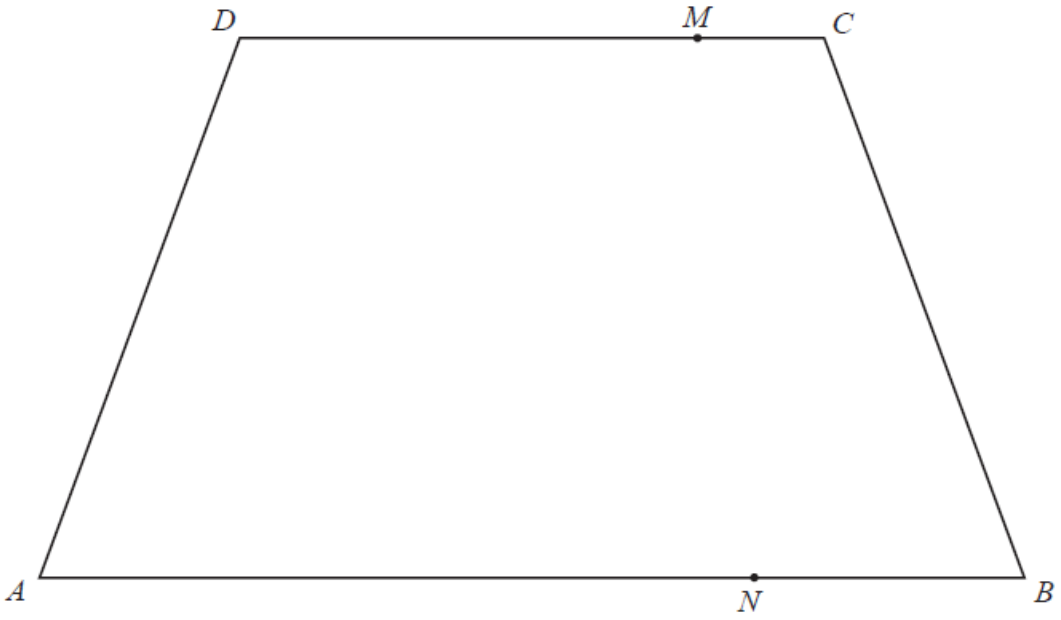
Plot the point C on the grid. [1]

(b) $\mathbf{p} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $\mathbf{q} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$.

Write $2\mathbf{p} - \mathbf{q}$ as a column vector.

$\begin{pmatrix} 0 \\ 7 \end{pmatrix}$ [2]

11.



NOT TO SCALE

$ABCD$ is a trapezium.
$AB = 2DC$, $DM = 2MC$ and $AN = 3NB$.
$\overrightarrow{AB} = \mathbf{p}$ and $\overrightarrow{AD} = \mathbf{q}$.

(a) Write $\overrightarrow{MC}$ in terms of $\mathbf{p}$.

$\frac{1}{6}\mathbf{p}$ [2]

(b) Find $\overrightarrow{MN}$ in terms of $\mathbf{p}$ and $\mathbf{q}$.

$\frac{5}{12}\mathbf{p} - \mathbf{q}$ [2]

12. $\mathbf{a} = \begin{pmatrix} 5 \\ -12 \end{pmatrix}$ $\mathbf{b} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

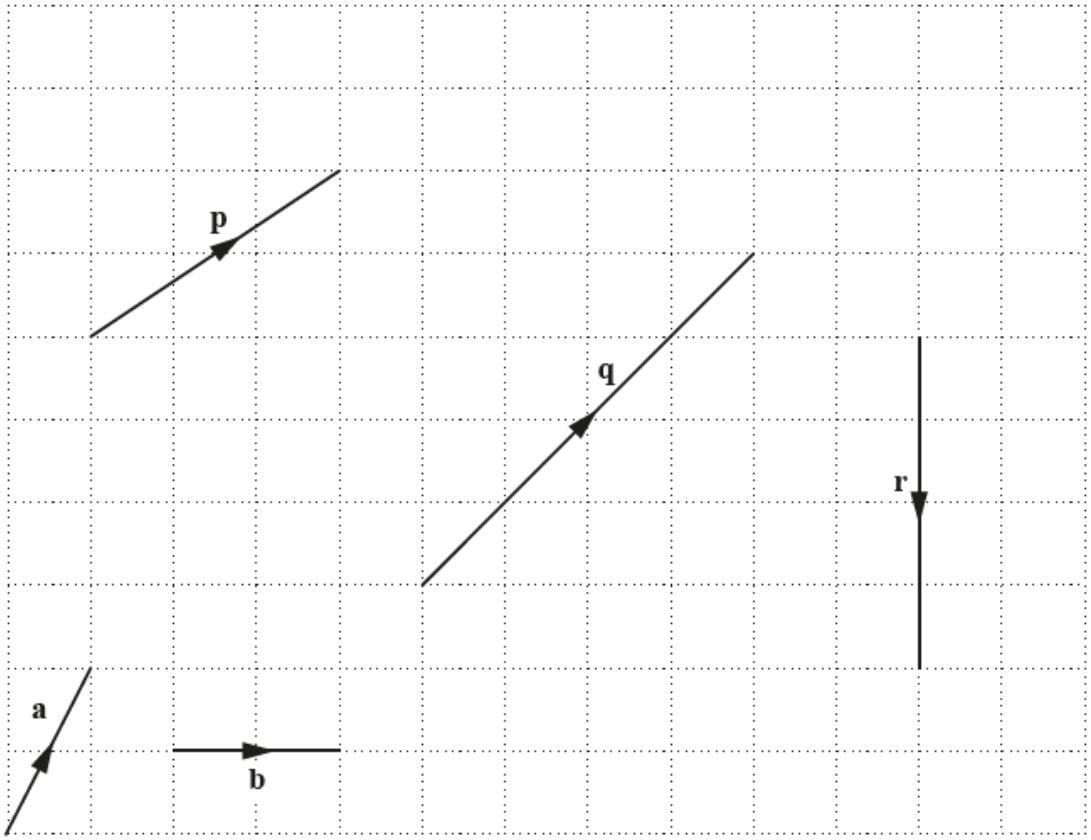
(a) Find $\mathbf{a} - 3\mathbf{b}$.

$\begin{pmatrix} -1 \\ -3 \end{pmatrix}$ [2]

(b) Work out $|\mathbf{a}|$.

$|\mathbf{a}| = 13$ [2]

13.



Write the vectors **p**, **q** and **r** in terms of **a** and **b**.

p = **a + b**

q = **2a + b**

r = **– 2a + b** [3]

14. $\mathbf{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\mathbf{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\mathbf{a} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}$

(a) $\mathbf{a} = p\mathbf{i} + q\mathbf{j}$

Find the values of p and q.

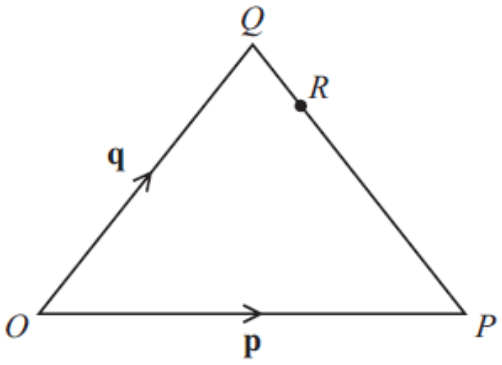
p = 4

q = – 6 [2]

(b) Calculate $|\mathbf{a}|$, giving your answer in the form $m\sqrt{n}$ where m and n are prime numbers.

$|\mathbf{a}|$ = $2\sqrt{13}$ [3]

15.



NOT TO SCALE

The diagram shows the vectors $\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OQ} = \mathbf{q}$.
R is on QP such that $QR = \frac{1}{4}QP$.

Find the following vectors in terms of **p** and **q**.
Give each answer in its simplest form.

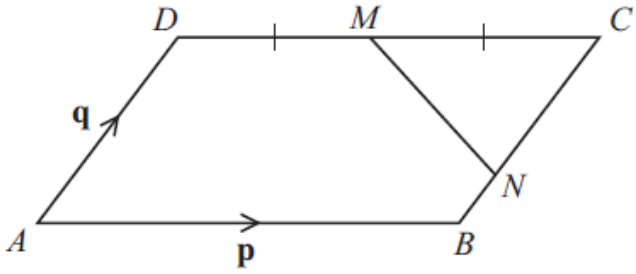
(a) $\overrightarrow{PQ}$

$\overrightarrow{PQ} =$ **q − p** ..... [1]

(b) $\overrightarrow{OR}$

$\overrightarrow{OR} =$ **$\frac{1}{4}$p + $\frac{3}{4}$q** ..... [2]

16.



NOT TO SCALE

$ABCD$ is a parallelogram.
$DM = MC$ and $CN = 2NB$.
$\overrightarrow{AB} = \mathbf{p}$ and $\overrightarrow{AD} = \mathbf{q}$.

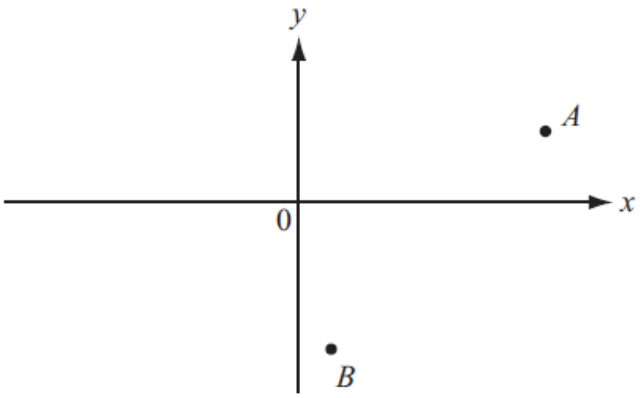
(a) Write down $\overrightarrow{CN}$ in terms of **q**.

$\overrightarrow{CN} =$ **$-\frac{2}{3}$q** ..... [1]

(b) Find $\overrightarrow{MN}$ in terms of **p** and **q**.

$\overrightarrow{MN} =$ **$\frac{1}{2}$p - $\frac{2}{3}$q** ..... [1]

17. **(a)**



NOT TO SCALE

A is the point (4, 2) and B is the point (1, – 3).

(i) Write down the vector $\overrightarrow{BA}$ in component form.

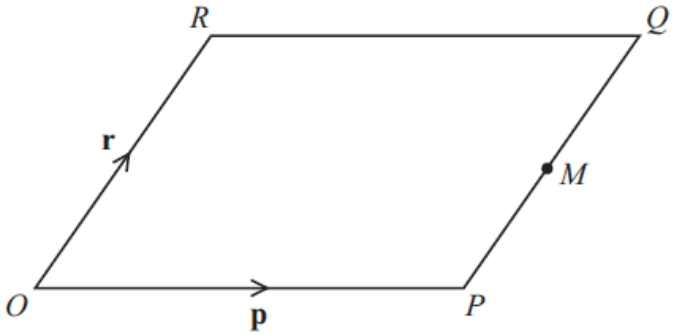
$\overrightarrow{BA} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ [1]

(ii) $\overrightarrow{BC} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$

Write down the co-ordinates of C.

(…– 2…… , …1……) [1]

(b)



NOT TO SCALE

$OPQR$ is a parallelogram and M is the midpoint of PQ.
$\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OR} = \mathbf{r}$.

Find $\overrightarrow{OM}$ in terms of $\mathbf{p}$ and $\mathbf{r}$.

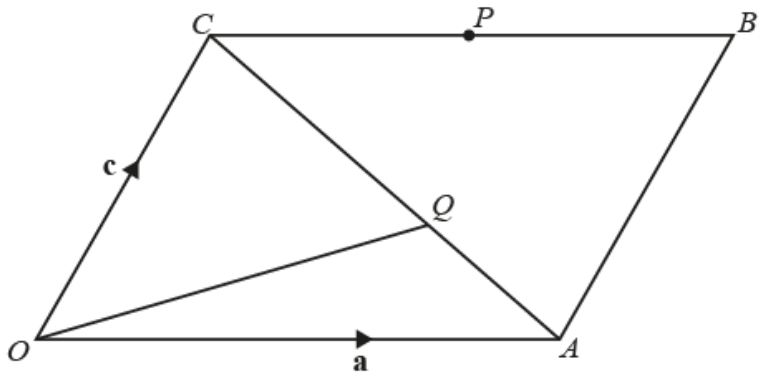
$\overrightarrow{OM}$ = …$\mathbf{p} + \frac{1}{2}\mathbf{r}$…… [2]

18. $\mathbf{p} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $\mathbf{q} = \begin{pmatrix} 1 \\ 6 \end{pmatrix}$

Find $2\mathbf{p} - 3\mathbf{q}$.

$\begin{pmatrix} 1 \\ -12 \end{pmatrix}$ …… [2]

19.



NOT TO SCALE

$OABC$ is a parallelogram.
P is the midpoint of CB.
$CQ : QA = 5 : 3$.

$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$.

Find these vectors in terms of **a** and/or **c**, giving your answers in their simplest form.

(a) $\overrightarrow{CP}$

$\overrightarrow{CP} =$ $\frac{1}{2}\mathbf{a}$ [1]

(b) $\overrightarrow{OQ}$

$\overrightarrow{OQ} =$ $\frac{5}{8}\mathbf{a} + \frac{3}{8}\mathbf{c}$ [3]

20.

$$p = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$$

Find $|p|$, giving your answer in the form $3\sqrt{a}$.

$|\boldsymbol{p}| =$ $3\sqrt{5}$ [2]

21. Q is the point (3, 7) and $\overrightarrow{PQ} = \begin{pmatrix} -6 \\ 3 \end{pmatrix}$.

(a) Find the co-ordinates of P.

(9 , 4) [2]

(b) Find $|\overrightarrow{PQ}|$.
Give your answer in its simplest surd form.

$3\sqrt{5}$ [3]

22. A is the point (1, 5) and B is the point (6, 2).

Find the column vector $\overrightarrow{AB}$.

$\begin{pmatrix} 5 \\ -3 \end{pmatrix}$ [2]

23. $\mathbf{a} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$ $\mathbf{b} = \begin{pmatrix} 2 \\ -8 \end{pmatrix}$

(a) Find $\mathbf{a} - 3\mathbf{b}$.

$\begin{pmatrix} 0 \\ 32 \end{pmatrix}$ [2]

(b) Work out $|\mathbf{a}|$.

10 [2]

24. $\mathbf{p} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ $\mathbf{q} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

(a) Find $\mathbf{p} + \mathbf{q}$.

$\begin{pmatrix} 4 \\ -3 \end{pmatrix}$ [1]

(b) A is the point (2, 7).
The point A is translated to the point B by the vector $\mathbf{p} + \mathbf{q}$.

Find the coordinates of B.

(6 , 4) [2]

25. (a) Work out $\begin{pmatrix} 12 \\ -5 \end{pmatrix} - 5\begin{pmatrix} 4 \\ -1 \end{pmatrix}$.

$\begin{pmatrix} -8 \\ 0 \end{pmatrix}$ [2]

(b) Work out the magnitude of $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$.

5 [2]

26. Work out $4 \times \begin{pmatrix} 6 \\ -2 \end{pmatrix}$.

$\begin{pmatrix} 24 \\ -8 \end{pmatrix}$ [1]

27. Find the magnitude of the vector $\begin{pmatrix} 2 \\ 6 \end{pmatrix}$.

Give your answer in simplest surd form.

$2\sqrt{10}$ [2]

28. $\mathbf{a} = \begin{pmatrix} 4 \\ -10 \end{pmatrix}$ $\mathbf{b} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$

Find the magnitude of the vector $\mathbf{a} - \mathbf{b}$.
Give your answer in its simplest surd form.

$4\sqrt{13}$ [4]

29. A is the point (1, 3) and B is the point (4, 9).

Find $\overrightarrow{AB}$.

$\overrightarrow{AB} = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$ [2]

30. $\mathbf{p} = \begin{pmatrix} -3 \\ 5 \end{pmatrix}$

(a) Find the column vector $3\mathbf{p}$.

$\begin{pmatrix} -9 \\ 15 \end{pmatrix}$ [1]

(b) Find $|\mathbf{p}|$, giving your answer in surd form.

$\sqrt{34}$ [2]

31. $\mathbf{p} = \begin{pmatrix} 12 \\ -5 \end{pmatrix}$

Find

(a) $2\mathbf{p}$,

$\begin{pmatrix} 24 \\ -10 \end{pmatrix}$ [1]

(b) $|\mathbf{p}|$.

13 [2]

32. $\mathbf{a} = \begin{pmatrix} -4 \\ -3 \end{pmatrix}$ $\mathbf{b} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

(a) Find $\mathbf{a} - 3\mathbf{b}$.

$\begin{pmatrix} -10 \\ 0 \end{pmatrix}$ [2]

(b) Find the magnitude of $\begin{pmatrix} -4 \\ -3 \end{pmatrix}$.

5 [2]