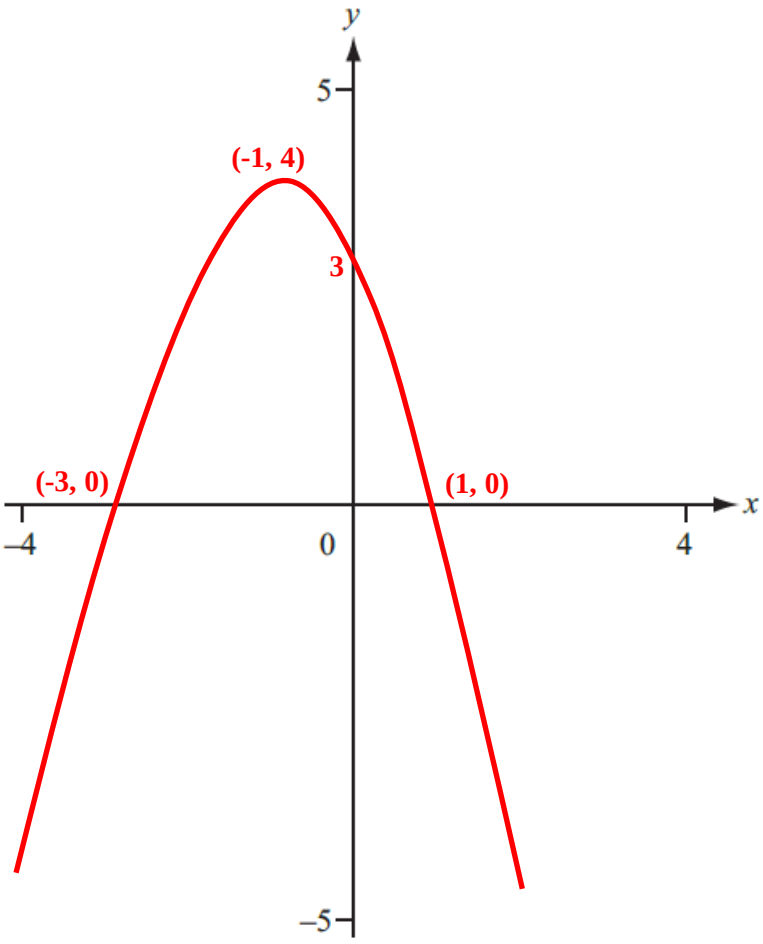
3 – Functions

Student name: Answers Score: _____

1. The graph of $y = f(x)$ where $f(x) = ax^2 + bx + 3$ crosses the x-axis at $(-3, 0)$ and $(1, 0)$.
The y coordinate of the vertex is 4.

(a) On the axes, sketch the graph of $y = f(x)$, for $-4 \leqslant x \leqslant 4$.



[2]

(b) Find the values of a and b.

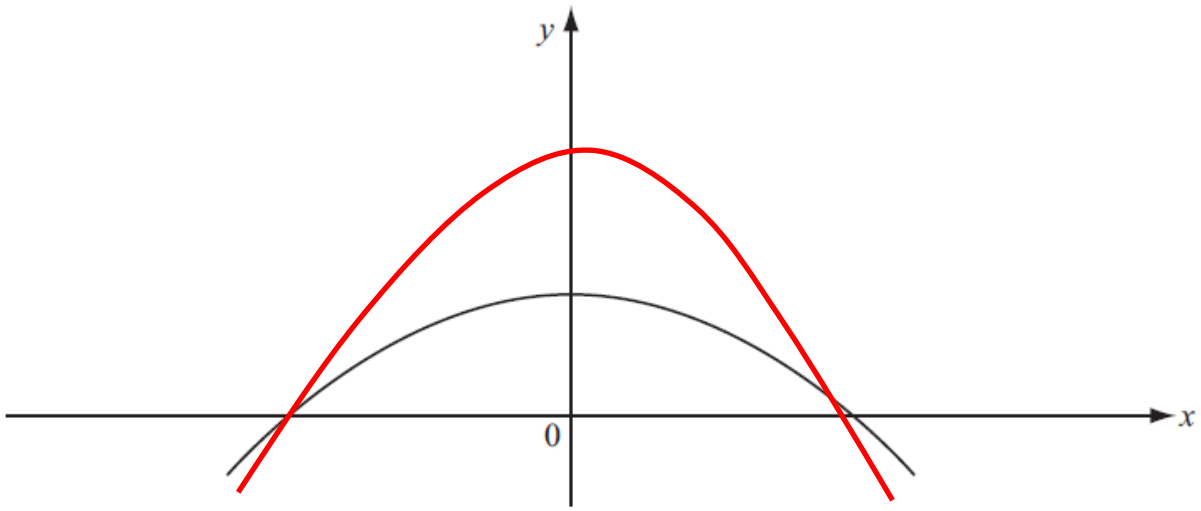
$a =$ -1 [1]

$b =$ -2 [1]

(c) Write down the range of $f(x)$ when the domain is $\mathbb{R}$.

$f(x) \leq 4$ [1]

2.



The sketch shows the graph of $y = f(x)$.

Using the same axes, sketch the graph of $y = 2f(x)$.

[2]

3.

$f(x) = 3 + 2x$

Find

(a) $f(f(-4))$,

-7 ..... [2]

(b) $f^{-1}(x)$.

$\frac{x-3}{2}$ ..... [2]

4.

$f(x) = x + 3$ and $g(x) = \frac{12}{x}$, $x \neq 0$

Find

(a) $g(f(1))$,

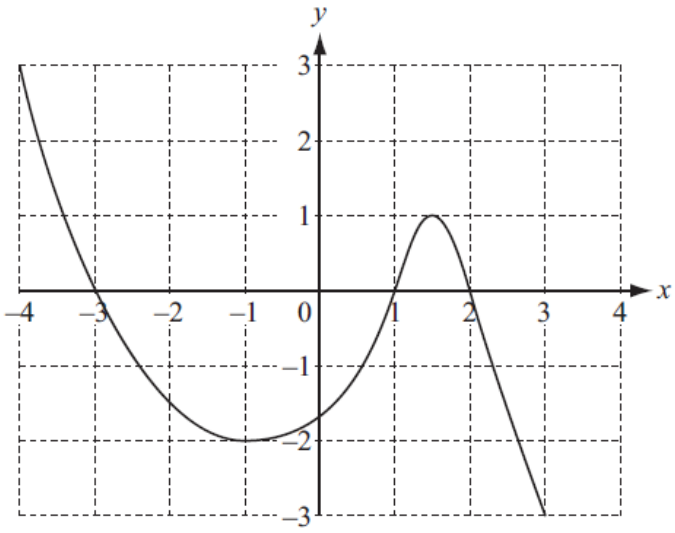
3 ..... [2]

(b) $g^{-1}(x)$.

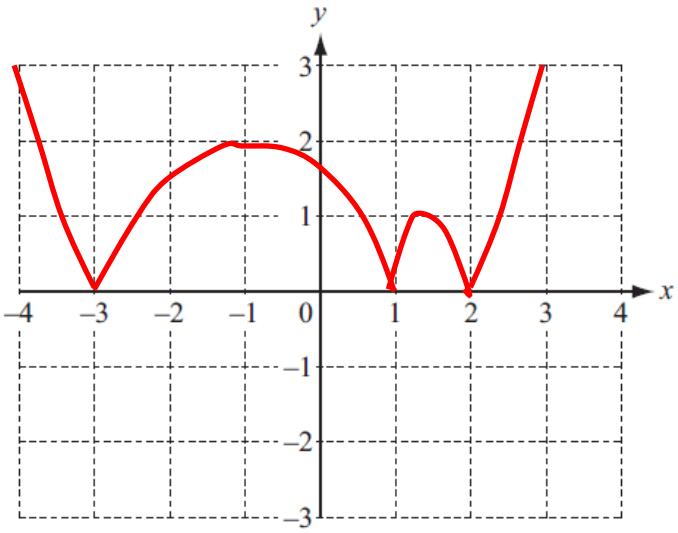
$\frac{12}{x}$ ..... [1]

5.

The diagram shows the graph of $y = f(x)$ for $-4 \leqslant x \leqslant 3$.

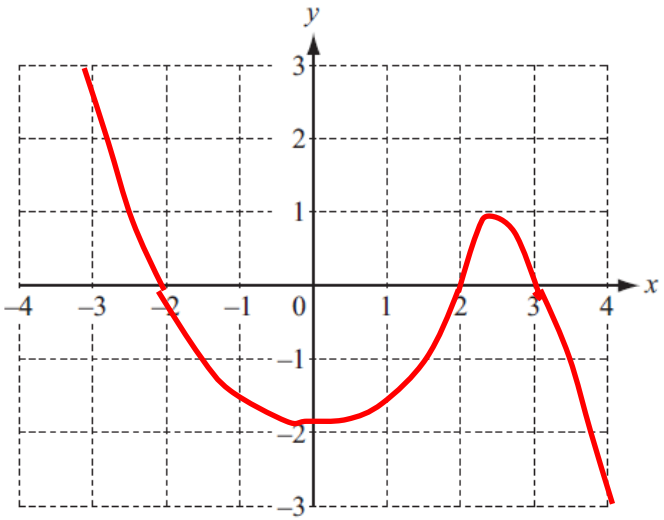


(a) On the diagram below, sketch the graph of $y = |f(x)|$.



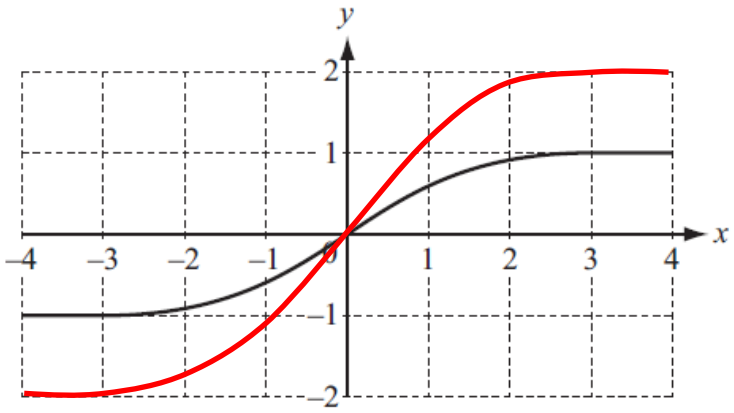
[3]

(b) On the diagram below, sketch the graph of $y = f(x - 1)$.



[2]

6. (a)

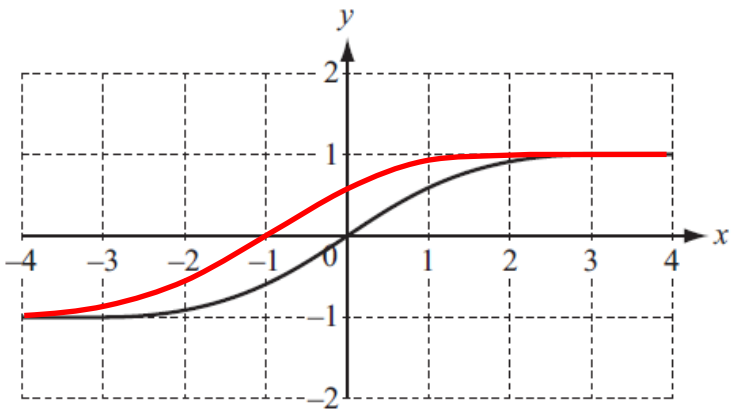


The diagram shows the graph of $y = f(x)$.

On the same diagram, sketch the graph of $y = 2f(x)$.

[1]

(b)



The diagram shows the graph of $y = f(x)$.

On the same diagram, sketch the graph of $y = f(x + 1)$.

[1]

7. $f(x) = \dfrac{1}{3x - 2}$

(a) Find $f(4)$.

0.1 ..... [1]

(b) Solve $f(x) = \dfrac{1}{4}$.

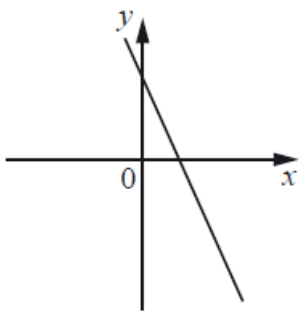
2 ..... [2]

(c) Find $f^{-1}(x)$.

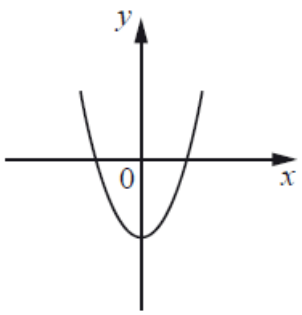
$\dfrac{1}{3}\left(\dfrac{1}{x}+2\right)$ ..... [3]

8. These are sketches of the graphs of six functions.

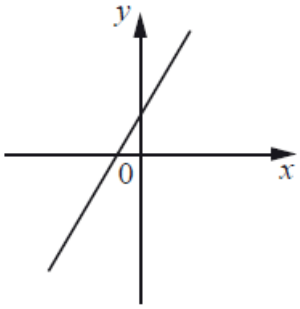
A



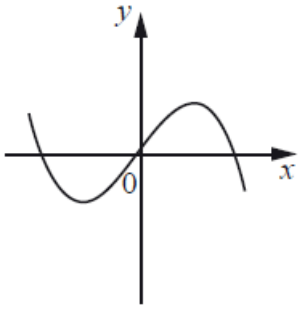
B



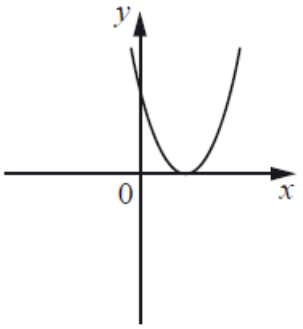
C



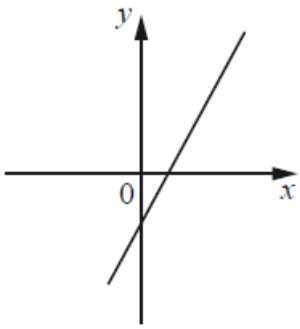
D



E



F

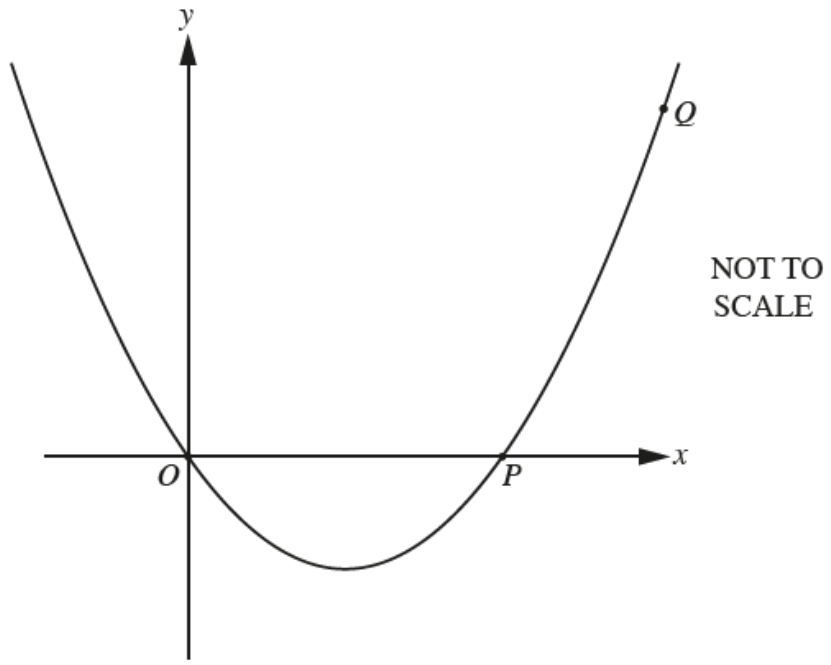


In the table below are four functions.
Write the correct letter in the table to match each function with its graph.

Function	Graph
$f(x) = 2x - 3$	F
$f(x) = (x-2)^2$	E
$f(x) = 4x - x^3$	D
$f(x) = 5 - 2x$	A

[4]

9.



The diagram shows a sketch of the graph of $y = ax^2 + bx$.
O is the point (0, 0), P is the point (4, 0) and Q is the point (8, 96).

Find the value of a and the value of b.

a = ..3.....

b = ..–12..... [3]

10.

$f(x) = 2x - 1$ $\quad$ $g(x) = 3x^2 + 1$

Find

(a) g(2),

.....13..... [1]

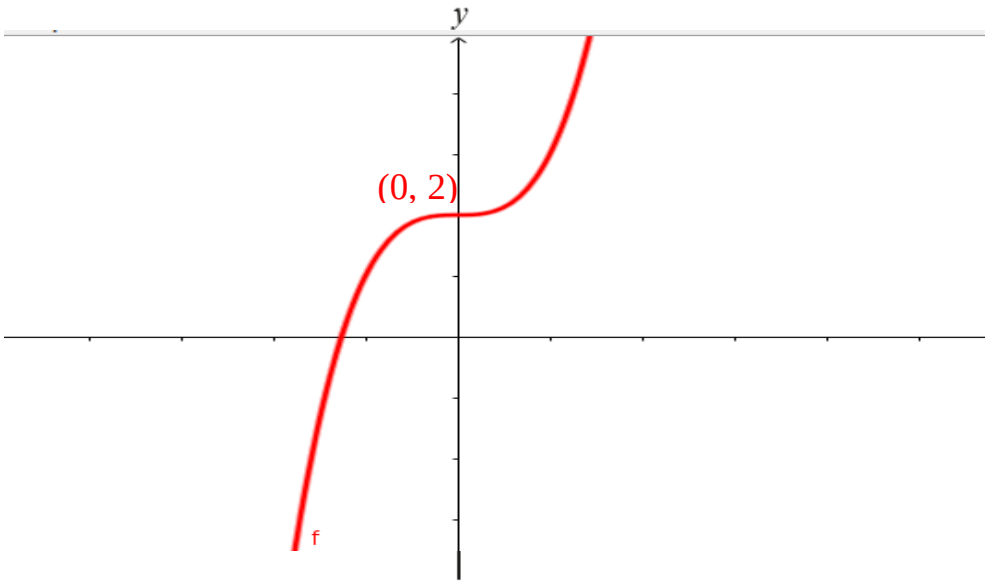
(b) g(f(x)),

.....$3(2x - 1)^2 + 1$..... [2]

(c) the inverse function $f^{-1}(x)$.

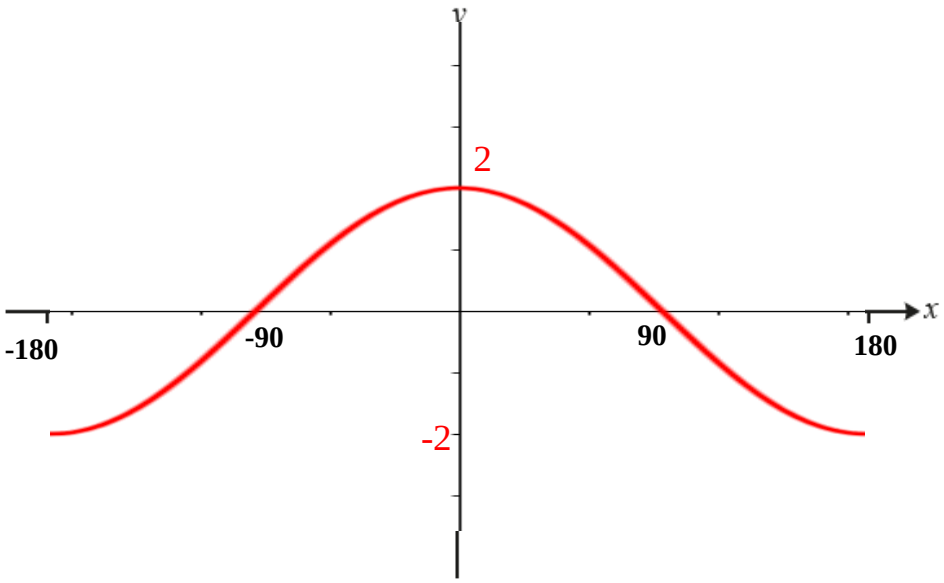
.....$\frac{x+1}{2}$..... [2]

11. (a) Sketch the graph of $y = x^3 + 2$.
Give the co-ordinates of the point where the graph crosses the y-axis.



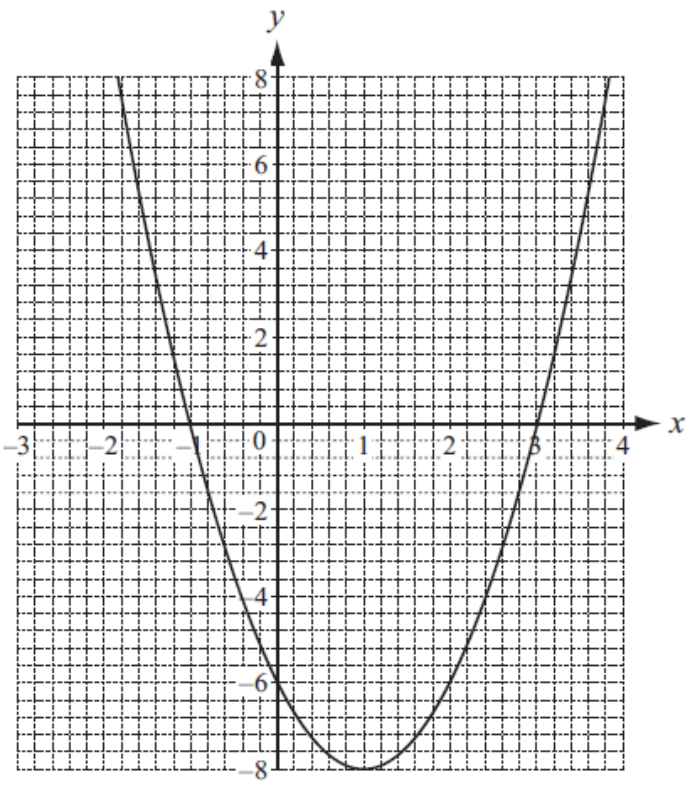
(0 , 2) [2]

(b) Sketch the graph of $y = 2\cos x$ for $-180° \leqslant x \leqslant 180°$.
Give the co-ordinates of the point where the graph crosses the y-axis.



(0 , 2) [3]

12. The diagram shows part of the graph of $y = f(x)$, where $f(x) = ax^2 + bx - 6$.



Find the values of a and b.

Answer $a =$ 2

$b =$ -4 [3]

13. $f(x) = 3x - 1$ $g(x) = 12 - x$

Find

(a) $f(g(8))$,

11 [2]

(b) $f(g(x))$, in its simplest form,

$35 - 3x$ [2]

(c) $g^{-1}(x)$.

$12 - x$ [1]

14. Which of the following functions are shown by the graphs below?
In each case $k > 1$.

Write the correct letter under each graph.

A $y = \dfrac{k}{x}$

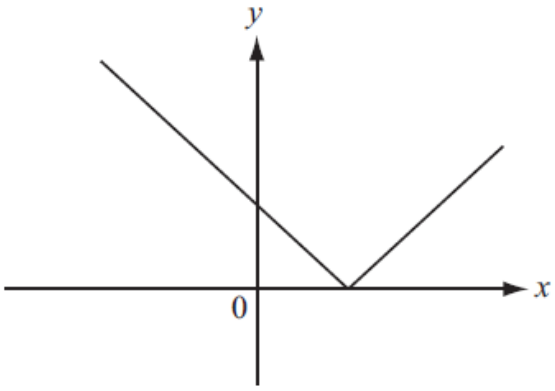
B $y = |x + k|$

C $y = k^x$

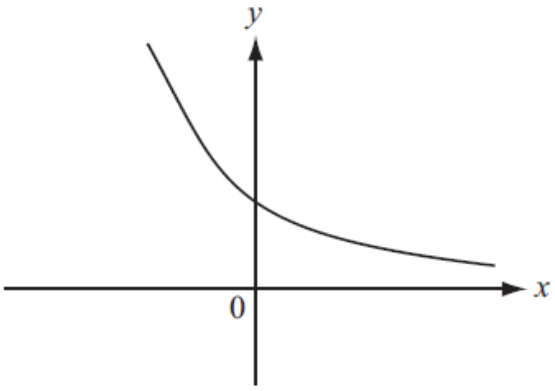
D $y = |x - k|$

E $y = k^{-x}$

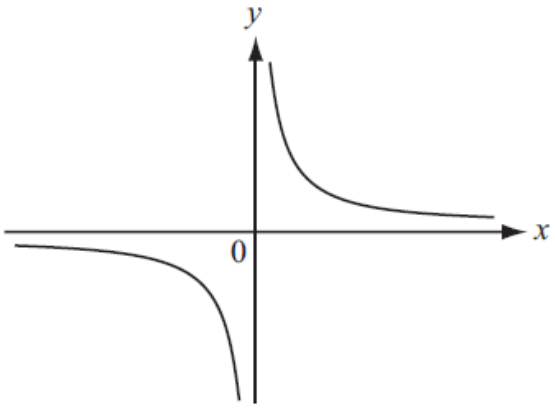
F $y = \dfrac{x}{k}$



D



E

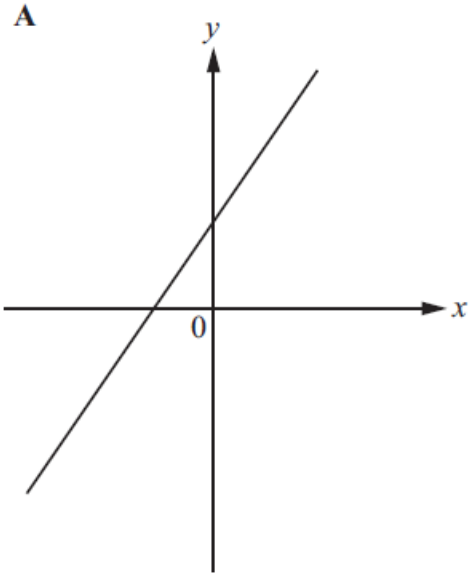


A

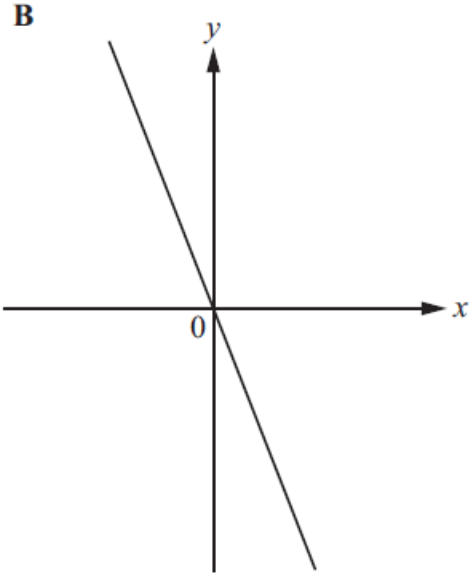
[3]

15. Here are the sketches of four graphs.

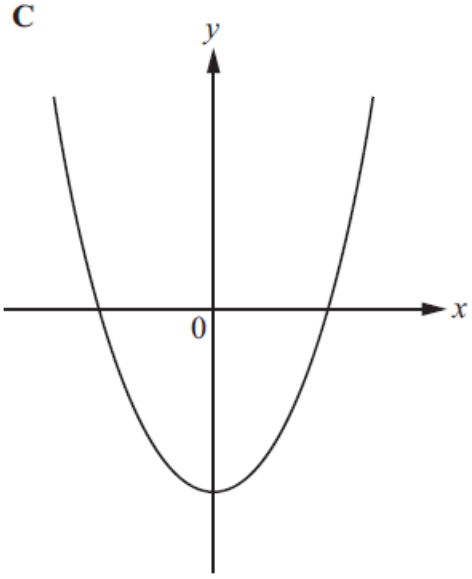
A



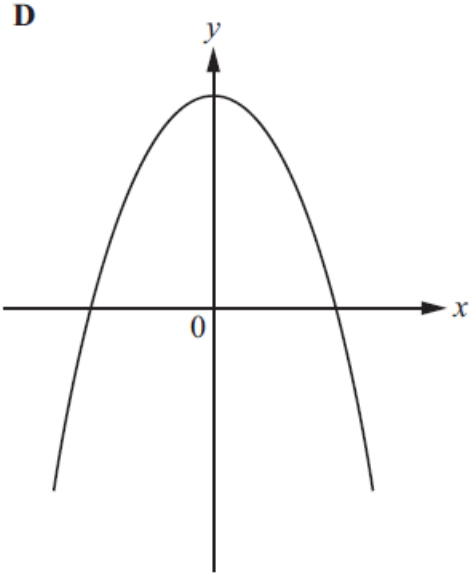
B



C



D



Each of the graphs represents one of these equations.

$y = x^2 + 3x$	$y = 3 - 2x$	$y = 3 - x^2$	$y = 2x + 3$
$y = x^2$	$y = -3x$	$y = x^2 - 3$	$y = 3x$

From the equations above, write down which one represents each graph.

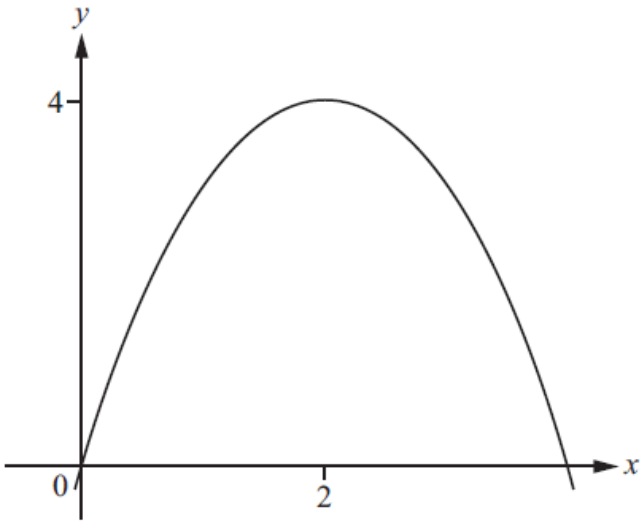
Answer Graph A $y =$ $y = 2x + 3$

Graph B $y =$ $y = -3x$

Graph C $y =$ $y = x^2 - 3$

Graph D $y =$ $y = 3 - x^2$ [4]

16.



NOT TO SCALE

The diagram shows the graph of $y = ax^2 + bx + c$.
The graph passes through (0, 0) and has a maximum point (2, 4).

Find the values of a, b and c.

a = ... – 1 .....

b = ... 4 .....

c = ... 0 ..... [3]

17.

$f(x) = 2x + 3$ $\quad$ $g(x) = 5 - 3x$

(a) Find $g(x)$ when $f(x) = 11$.

..... – 7 ..... [2]

(b) Find and simplify an expression for $f(g(x))$.

..... 13 – 6x ..... [2]

(c) Find $g^{-1}(x)$.

..... $\frac{5-x}{3}$ ..... [2]

18.

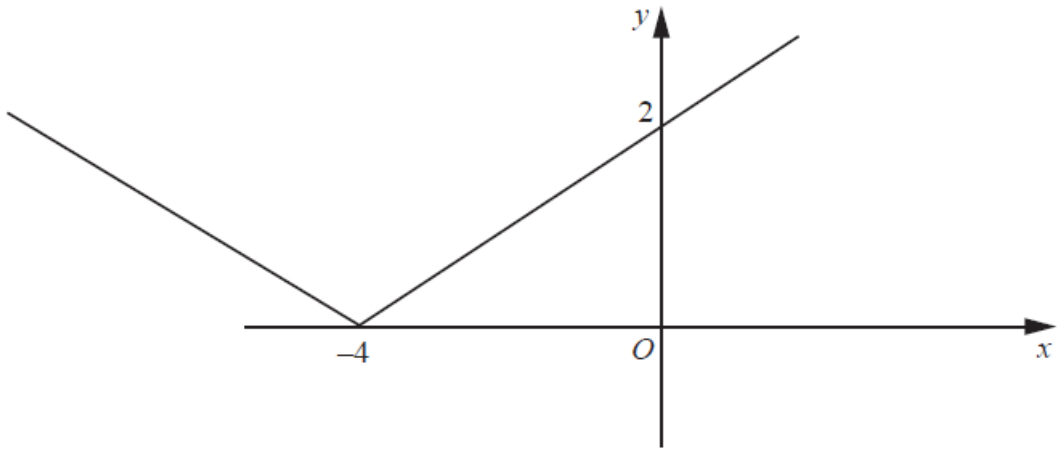
The graph of $y = (x - h)^2 + k$ has a vertex at $(2, -3)$.

Find the value of h and the value of k.

h = ... 2 .....

k = ... – 3 ..... [2]

19.



NOT TO SCALE

The diagram shows the graph of $y = |px + q|$.

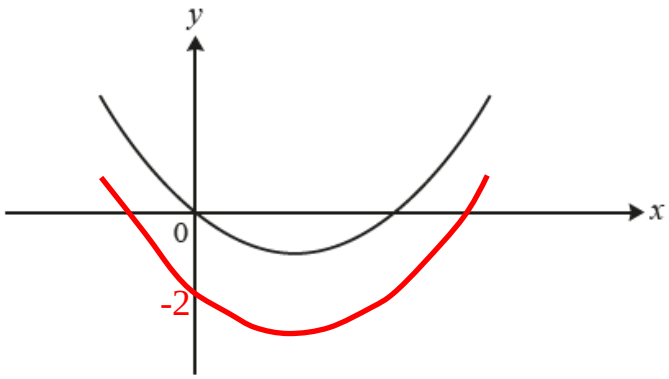
Find the value of p and the value of q.

p = ……0.5……..

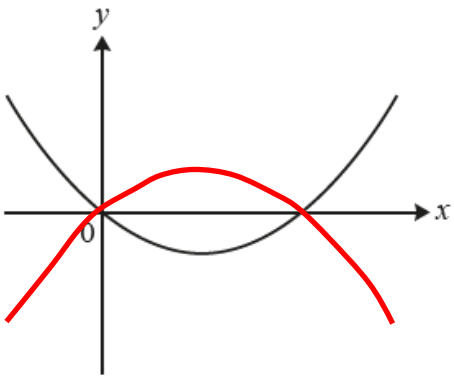
q = ……2…….. [2]

20. Each diagram shows the graph of $y = f(x)$.

On each diagram, sketch the function indicated.



$y = f(x) - 2$



$y = -f(x)$

[2]

21. $g(x) = \dfrac{2x+1}{x-1}, \quad x \neq 1$

Solve the equation $g^{-1}(x) = 2$.

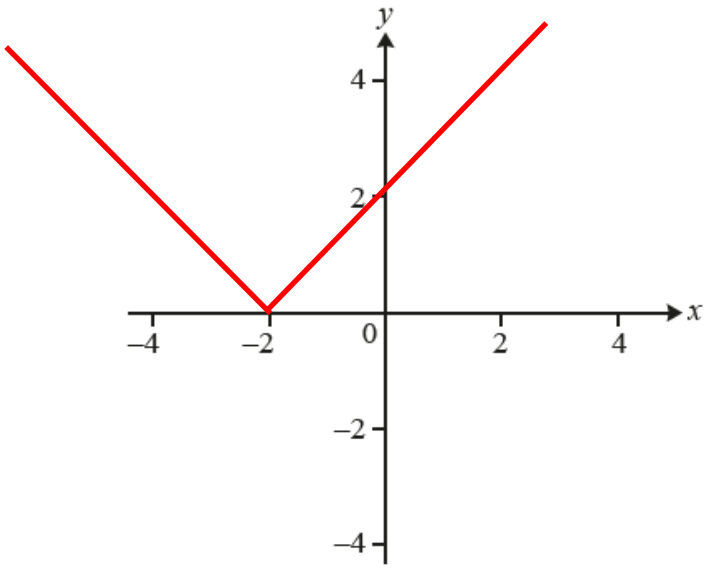
x = ……5…….. [1]

22. $f(x) = 10^x$

Find $f^{-1}(x)$.

$f^{-1}(x)$ = ……log x…….. [1]

23. Sketch the graph of $y = |x+2|$.



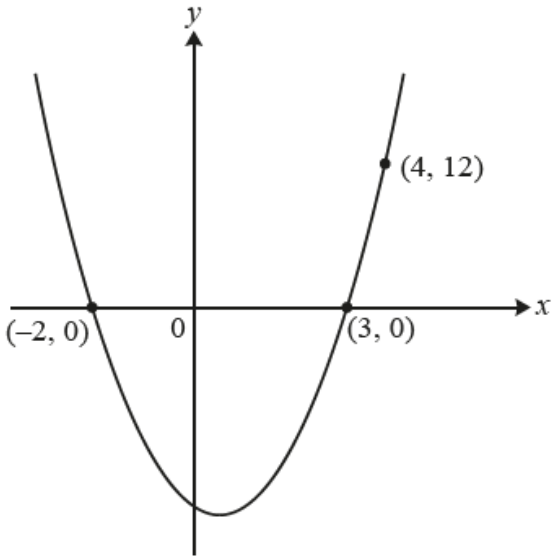
[3]

24. $f(x) = x^3 - 2$

Find the value of x when $f(x) = 25$.

$x =$ 3 ..... [2]

25.



NOT TO SCALE

The equation of this curve is $y = ax^2 + bx + c$.
Find the values of a, b and c.

$a =$ 2 .....

$b =$ − 2 .....

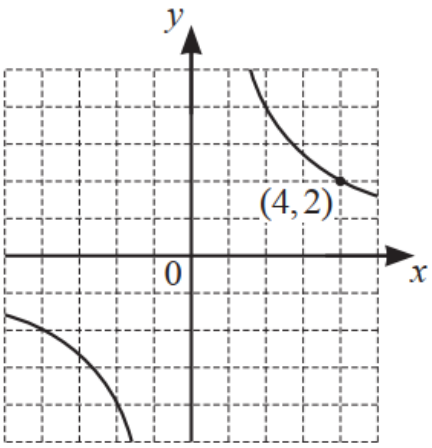
$c =$ − 12 ..... [3]

26. $f(x) = 1 - 3x$

Find the value of $f(-1)$.

4 ..... [1]

27.

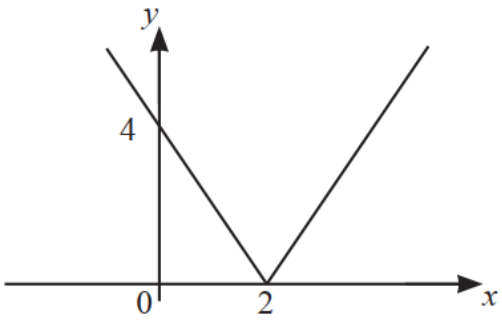


In the diagram, the graph passes through the point (4, 2).

Write down the equation of the graph.

$y = \frac{8}{x}$ [2]

28.



NOT TO SCALE

The diagram shows the graph of $y = |ax + b|$, where $a > 0$.

Find the value of a and the value of b.

$a =$ 2

$b =$ –4 [2]

29. $f(x) = |2x + 3|$

Find the values of x when $f(x) = 15$.

$x = 6$ $x = -9$ [2]

30. $f(x) = 2x - 3$

Find the range of $f(x)$ for the domain $\{0, 1, 2\}$.

{–3, –1, 1} [1]

31. $f(x) = |2x - 7|$ for all real x.

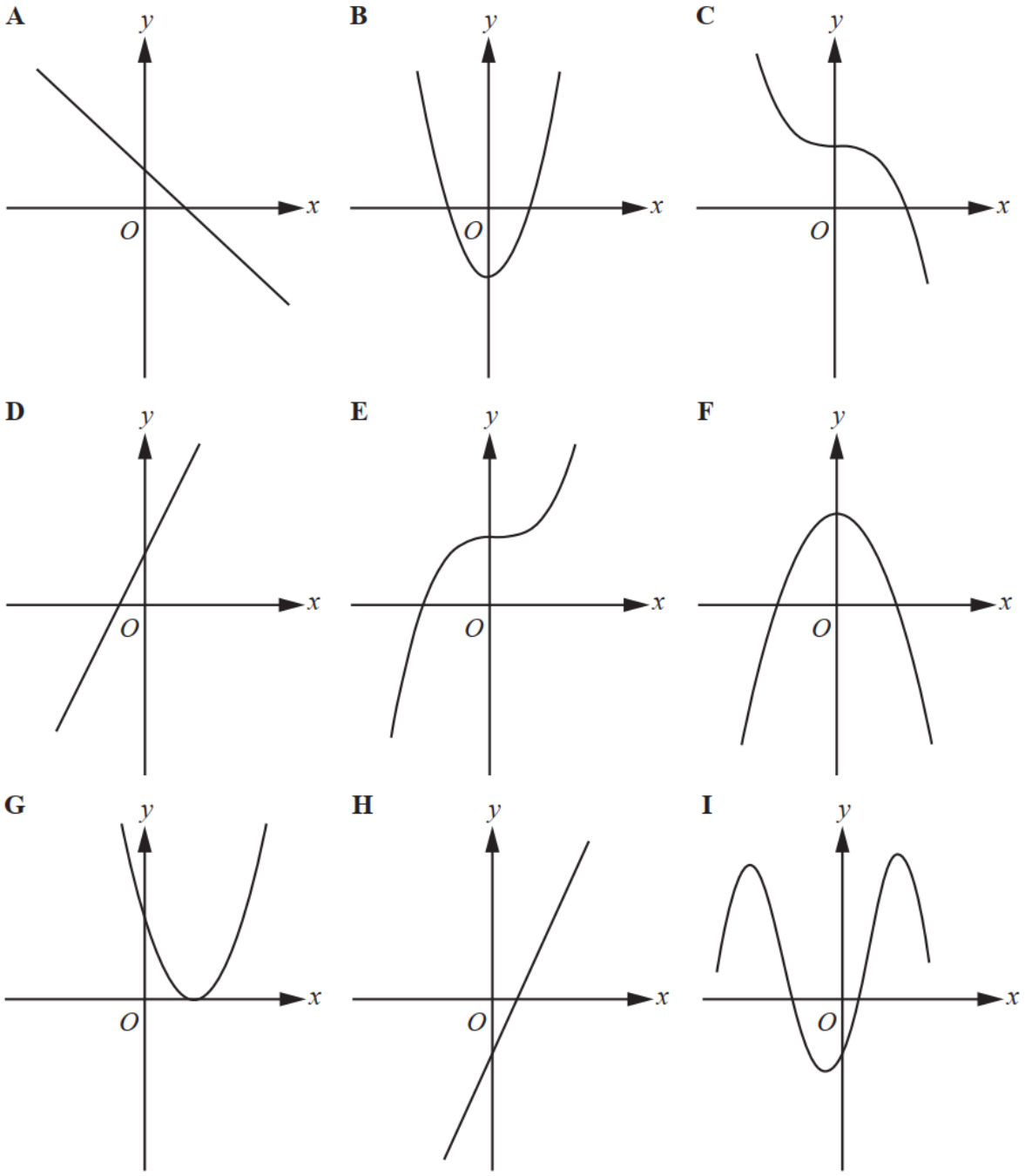
(a) Find $f(2)$.

3 [1]

(b) Write down the range of $f(x)$.

$f(x) \geq 0$ [1]

32. The diagram shows nine sketch graphs.



Write the letter of the graph which shows each of these functions.

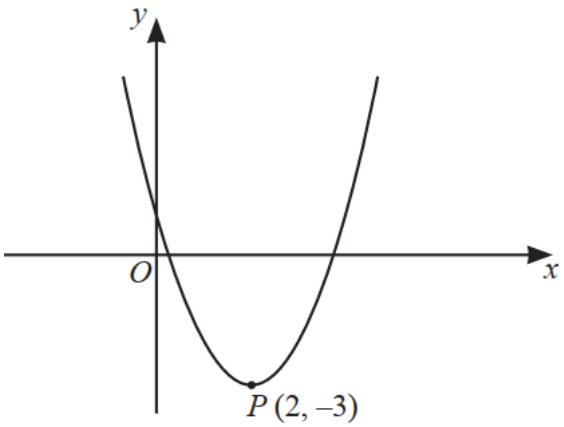
$f(x) = 2x - 3$ Graph H

$f(x) = x^2 - 3$ Graph B

$f(x) = 3 - x^3$ Graph C

$f(x) = (x-3)^2$ Graph G [4]

33.



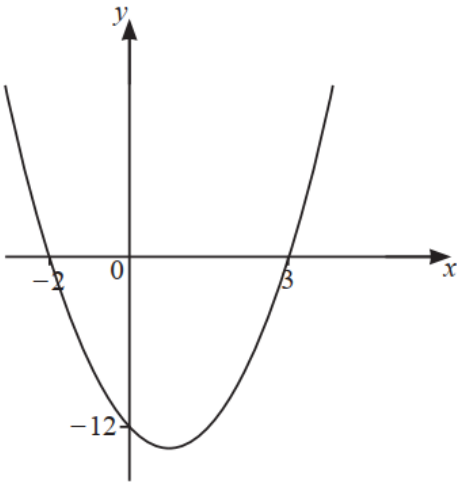
NOT TO SCALE

The diagram shows a sketch of the graph $y = x^2 + bx + c$.
The minimum point is at $P(2, -3)$.

Find the value of b and the value of c.

$b =$ –4 $c =$ 1 [3]

34.



NOT TO SCALE

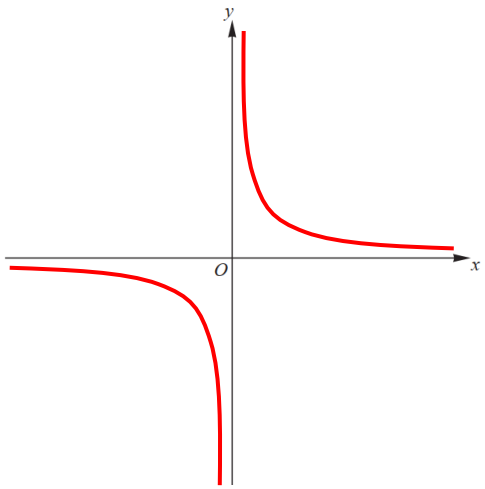
The equation of the curve is $y = ax^2 + bx - 12$.

Find the value of a and the value of b.

$a =$ 2

$b =$ –2 [3]

35. On the diagram, sketch the graph of $y = \frac{1}{x}$.



[2]

36. $f(x) = \frac{1}{2x-5}$, $x \neq 2.5$

(a) Find $f(2)$.

-1 ..... [1]

(b) Solve $f(x) = 5$.

$\frac{13}{5}$ ..... [2]

37. $f(x) = 11x+2$ $g(x) = \sin x°$

(a) Find $f^{-1}(x)$.

$f^{-1}(x) = \frac{x-2}{11}$ ..... [2]

(b) Find $g(f(8))$.

1 ..... [2]