Name: _____ Score: _____

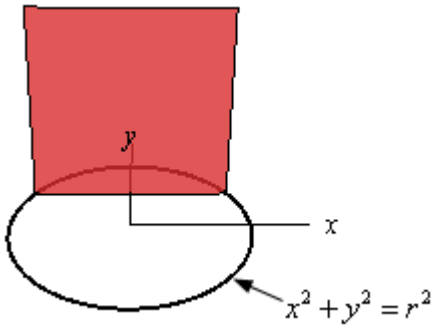
Teacher: _____ Date: _____

Volume of solids - Answers

1. Find the volume of a pyramid of height h whose base is an equilateral triangle of length L

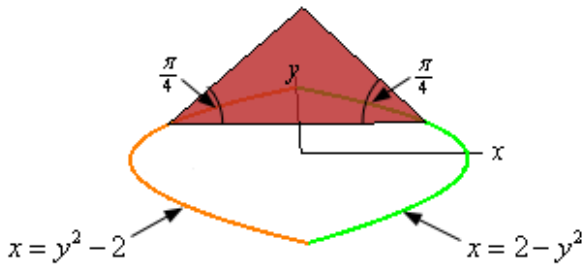
$$V = \int_0^h \frac{\sqrt{3}L^2}{4h^2} y^2\, dy = \frac{\sqrt{3}L^2}{4h^2} \int_0^h y^2\, dy = \frac{\sqrt{3}L^2}{4h^2} \left(\frac{1}{3}y^3\right)\Big|_0^h = \boxed{\frac{\sqrt{3}\,L^2 h}{12}}$$

2. Find the volume of the solid whose base is a disk of radius r and whose cross-sections are squares. See figure below to see a sketch of the cross-sections.



$$V = \int_{-r}^{r} 4\left(r^2 - y^2\right)\, dy = 4\left(yr^2 - \frac{1}{3}y^3\right)\Big|_{-r}^{r} = \boxed{\frac{16}{3}r^3}$$

3. Find the volume of the solid whose base is the region bounded by $x = 2 - y^2$ and $x = y^2 - 2$ and whose cross-sections are isosceles triangles with the base perpendicular to the y-axis and the angle between the base and the two sides of equal length is $\frac{\pi}{4}$. See figure below to see a sketch of the cross-sections.



$$V = \int_{-\sqrt{2}}^{\sqrt{2}} 4 - 4y^2 + y^4\, dy = \left(4y - \frac{4}{3}y^3 + \frac{1}{5}y^5\right)\Big|_{-\sqrt{2}}^{\sqrt{2}} = \boxed{\frac{64\sqrt{2}}{15}}$$