## Solution of systems of equations

Student name: \_\_\_\_\_\_ Score: \_\_\_\_\_

1. Let  $A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 0 & 1 \end{pmatrix}$ ,  $B = \begin{pmatrix} 18 \\ 23 \\ 13 \end{pmatrix}$  and  $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ .

Consider the equation AX = B.

- (i) Express X in terms of  $A^{-1}$  and B.
- (ii) Hence, solve for X.
- 2. Consider the simultaneous equations

$$x+2y = 7$$

$$-3x+y-z = 10$$

$$2x-2y+z = -12$$

- (a) Write these equations as a matrix equation.
- (b) Solve the matrix equation.
- 3. Consider the simultaneous equations

$$x-3y = 1$$
$$2x+z = 2$$
$$4x+y+3z = -1$$

- (a) Write these equations as a matrix equation.
- (b) Solve the matrix equation.

**4.** Let 
$$M = \begin{pmatrix} 2 & 1 \\ 2 & -1 \end{pmatrix}$$
.

solve 
$$M \binom{x}{y} = \binom{4}{8}$$
.

Let 
$$A = \begin{pmatrix} 5 & 1 \\ 6 & 2 \end{pmatrix}$$
,  $X = \begin{pmatrix} x \\ y \end{pmatrix}$  and  $C = \begin{pmatrix} 8 \\ -4 \end{pmatrix}$ . Solve the matrix equation  $AX = C$ .



**6.** A matrix M has inverse  $M^{-1} = \begin{pmatrix} 5 & 0 \\ 1 & 2 \end{pmatrix}$ .

Solve the matrix equation MX = B, where  $B = \begin{pmatrix} 1 \\ 7 \end{pmatrix}$  and  $X = \begin{pmatrix} x \\ y \end{pmatrix}$ .

7. Let 
$$A = \begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix}$$
 and  $B = \begin{pmatrix} -5 \\ 5 \end{pmatrix}$ .

Solve  $A^{-1}X = B$ .

**8.** Let 
$$A = \begin{pmatrix} 2 & -4 \\ -1 & 3 \end{pmatrix}$$
.

Solve the matrix equation  $AX = \begin{pmatrix} 4 & 6 \\ 2 & -2 \end{pmatrix}$ .

- **9.** Let A, B, C and X be square matrices, such that XA + B = C.
  - (a) Find an expression for X in terms of A, B and C.

(b) Given that 
$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$
,  $\mathbf{B} = \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}$  and  $\mathbf{C} = \begin{pmatrix} 1 & 3 \\ -3 & 4 \end{pmatrix}$ , find  $\mathbf{X}$ .

10. Let 
$$A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix}$$
 and  $B = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$  Solve  $AX = B$ .

**11.** Let 
$$\mathbf{A} = \begin{pmatrix} 0 & 5 & 4 \\ 1 & 2 & 1 \\ 2 & 2 & 0 \end{pmatrix}$$
, and  $\mathbf{B} = \begin{pmatrix} 11 \\ 7 \\ 10 \end{pmatrix}$ . Solve  $\mathbf{AX} = \mathbf{B}$ .

## Solution of systems of equations

Student name: \_\_\_\_\_ ANSWERS Score:

Let  $\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 0 & 1 \end{pmatrix}$ ,  $\mathbf{B} = \begin{pmatrix} 18 \\ 23 \\ 13 \end{pmatrix}$  and  $\mathbf{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ . 1.

Consider the equation AX = B.

- Express X in terms of  $A^{-1}$  and B. (i)
- **Hence**, solve for X. (ii)
- 2. Consider the simultaneous equations

$$x + 2y = 7$$

$$-3x + y - z = 10$$

$$2x - 2y + z = -12\begin{pmatrix} 1 & 2 & 0 \\ -3 & 1 & -1 \\ 2 & -2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ 10 \\ -12 \end{pmatrix}$$
ix equation.

- Write these equations as a matrix equation. (a)
- x = 4, y = 1, z = -6Solve the matrix equation.
- Consider the simultaneous equations

$$x-3y=1$$

$$2x + z = 2$$

$$4x + y + 3z = -1$$
Write these equations as a matrix equation. 
$$\begin{pmatrix} 1 & -3 & 0 \\ 2 & 0 & 1 \\ 4 & 1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

- (a)
- Solve the matrix equation. x = 4, y = 1, z = -6
- **4.** Let  $M = \begin{pmatrix} 2 & 1 \\ 2 & -1 \end{pmatrix}$ .

solve 
$$M \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$$
.  $x = 3, y = -2$ 

5. Let 
$$A = \begin{pmatrix} 5 & 1 \\ 6 & 2 \end{pmatrix}$$
;  $X = \begin{pmatrix} x \\ y \end{pmatrix}$  and  $C = \begin{pmatrix} 8 \\ -4 \end{pmatrix}$ . Solve the matrix equation  $AX = C$ .  $x = 5$ ,  $y = -17$ 



**6.** A matrix M has inverse  $M^{-1} = \begin{pmatrix} 5 & 0 \\ 1 & 2 \end{pmatrix}$ .

Solve the matrix equation MX = B, where  $B = \begin{pmatrix} 1 \\ 7 \end{pmatrix}$  and  $X = \begin{pmatrix} x \\ y \end{pmatrix}$ . x = 5, y = 15

7. Let  $A = \begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix}$  and  $B = \begin{pmatrix} -5 \\ 5 \end{pmatrix}$ .

Solve  $A^{-1}X = B$ .  $\begin{pmatrix} -15 \\ 5 \end{pmatrix}$ 

**8.** Let  $A = \begin{pmatrix} 2 & -4 \\ -1 & 3 \end{pmatrix}$ .

Solve the matrix equation  $AX = \begin{pmatrix} 4 & 6 \\ 2 & -2 \end{pmatrix}$ .  $\begin{pmatrix} 10 & 5 \\ 4 & 1 \end{pmatrix}$ 

**9.** Let A, B, C and X be square matrices, such that XA + B = C.

(a) Find an expression for X in terms of A, B and C.  $X = (C - B)A^{-1}$ 

(b) Given that 
$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$
,  $\mathbf{B} = \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}$  and  $\mathbf{C} = \begin{pmatrix} 1 & 3 \\ -3 & 4 \end{pmatrix}$ , find  $\mathbf{X}$ .  $\begin{pmatrix} 1 & 0 \\ 11 & -5 \end{pmatrix}$ 

10. Let 
$$A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix}$$
 and  $B = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$  Solve  $AX = B$ .  $\begin{pmatrix} -4 \\ 4 \\ -5 \end{pmatrix}$ 

**11.** Let 
$$\mathbf{A} = \begin{pmatrix} 0 & 5 & 4 \\ 1 & 2 & 1 \\ 2 & 2 & 0 \end{pmatrix}$$
, and  $\mathbf{B} = \begin{pmatrix} 11 \\ 7 \\ 10 \end{pmatrix}$ . Solve  $\mathbf{AX} = \mathbf{B}$ .